Supplement for Safe Task Space Synchronization with Time-Delayed Information

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In this supplement, bounds on certain terms of the Lyapunov function derivative are computed. The results of these bounds are used in the main paper.

Appendix A
Development of a bounds on $\dot{P}_1$, $\dot{P}_2$ and $\dot{P}_3$

The time derivatives of the LK functional terms using Leibniz rule yield

$$\dot{P}_1 \leq Tc_1 - \frac{K_{LK_1}\omega_1}{2}\int_{t_T}^{t} \dddot{p}_h^T(l)\dddot{p}_h(l)dl$$

where $c_1 \geq \frac{K_{LK_1}\omega_1}{2}\|\dddot{p}_h(t)\|^2 \in \mathbb{R}^+$ is a constant. Similarly, the derivative of P_2 can be computed as follows

$$\dot{P}_2 = \frac{K_{LK_2}\omega_2 T}{2}\|\dot{\phi}(e_p)e_p(t) + \phi(e_p)(t)\dot{e}_p(t)\|^2 - \frac{K_{LK_2}\omega_2}{2}\int_{t_T}^{t}[\frac{d}{dt}(\phi(e_p)(l)e_p(l))]^T[\frac{d}{dt}(\phi(e_p)(l)e_p(l))]dl$$

where $\phi(e_p)$, and $\dot{\phi}(e_p)$ denote diagonal matrices containing terms $\phi(e_{pi})$, and $\dot{\phi}(e_{pi})$, $\forall i = \{1,..,n\}$, respectively. Substituting for $\dot{\phi}$, $\dot{\eta}$ and $\dot{e}_p$, using triangle and Young's inequalities, the following upper bound on the first term of $\dot{P}_2$ is obtained

$$\begin{aligned} &k_1^2 6K_{LK_2}\omega_2 T\|\phi^3(e_p)e_p^2\|^2\|e_p\|^2 \\ &+ 6K_{LK_2}\omega_2 T w_j^2\|\phi^2(e_p)e_p^2\|^2\|\dot{\theta}\|^2\|\tilde{\zeta}_j\|^2 \\ &+ 6K_{LK_2}\omega_2 T j_p^2\|\phi^2(e_p)e_p^2\|^2\|\eta\|^2 \\ &+ k_1^2 6K_{LK_2}\omega_2 T\|\phi^2(e_p)e_p\|^2 \\ &+ 6K_{LK_2}\omega_2 T w_j^2\|\phi(e_p)\|^2\|\dot{\theta}\|^2\|\tilde{\zeta}_j\|^2 \\ &+ 6K_{LK_2}\omega_2 T j_p^2\|\phi(e_p)\|^2\|\eta\|^2 \leq T\rho_1(\|y\|)\|y\|^2 \end{aligned} \quad (1)$$

where $y = [e_p^T \quad \eta^T \quad \tilde{\zeta}_j^T]^T$, $\rho_1(\|y\|)$ is a positive invertible non-decreasing function. The symbol $\phi^3(e_p)e_p^2 = [\phi^3(e_{p1})e_{p1}^2, ..., \phi^3(e_{pn})e_{pn}^2]^T$ is a vector containing elements $\phi^3(e_{pi})e_{pi}^2$, $\phi^2(e_{pi})e_{pi}^2 = [\phi^2(e_{p1})e_{p1}^2, ..., \phi^2(e_{pn})e_{pn}^2]^T$ is a vector containing elements of $\phi^2(e_{pi})e_{pi}^2$, and $\phi^2(e_p)e_p$ is a vector containing elements $\phi^2(e_{pi})e_{pi}$.

$\dot{P}_3$ is computed as

$$\dot{P}_3 \leq Tc_2 - \frac{K_{LK_3}\omega_3}{2}\int_{t_T}^{t} \dot{p}_h^T(l)\dot{p}_h(l)dl$$

where $c_2 \geq \frac{K_{LK_3}\omega_3}{2}\|\dot{p}_h(t)\|^2 \in \mathbb{R}^+$ is a constant.

Appendix B
Development of a bound on $\eta^T(W_y\zeta - W_{yT}\zeta)$

Consider following term from (34) (of the main paper)

$$\begin{aligned} \eta^T(W_y - W_{y_T})\zeta_y = M[&\dot{\hat{J}}_p^{-1}(\dot{p}_h - \dot{p}_{h_T}) \\ &+ k_1\dot{\hat{J}}_p^{-1}(\phi(e_p)e_p - \phi(e_{p_T})e_{p_T}) \\ &+ \hat{J}_p^{-1}(\ddot{p}_h - \ddot{p}_{h_T}) + k_1\hat{J}_p^{-1}(\dot{\phi}(e_p)e_p \\ &- \dot{\phi}(e_{p_T})e_{p_T}) + k_1\hat{J}_p^{-1}(\phi(e_p)\dot{e}_p - \phi(e_{p_T})\dot{e}_{p_T})] \\ &+ C(\theta,\dot{\theta})(\eta - \eta_T) \end{aligned} \quad (2)$$

The norm bound can be written as

$$\begin{aligned} &\|\eta\|\|(W_y - W_{y_T})\zeta_y\| \leq \bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}\|\dot{\theta}\|\|\eta\|\|\int_{t_T}^{t}\ddot{p}_h(l)dl\| \\ &+ k_1\bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}\|\dot{\theta}\|\|\eta\|\|\int_{t_T}^{t}\frac{d}{dl}\phi(e_p)(l)e_p(l)dl\| \\ &+ k_1\bar{m}\bar{j}_{p_{inv}}\|\eta\|\|\dot{\phi}(e_p)e_p - \dot{\phi}(e_{p_T})e_{p_T} + \phi(e_p)\dot{e}_p \\ &- \phi(e_{p_T})\dot{e}_{p_T}\| + \bar{c}\|\dot{\theta}\|\|\eta\|\|\eta - \eta_T\| + \bar{m}\bar{j}_{p_{inv}}\bar{p}_3\|\eta\| \end{aligned} \quad (3)$$

where $\|C(\theta,\dot{\theta})\| \leq \bar{c}\|\dot{\theta}\|$ is used. Using Young's inequality on select terms, and the following bound on $\|\eta - \eta_T\|$ developed using (7) and (9) (of the main paper)

$$\|\eta - \eta_T\|^2 \leq 2j_{p_i}^2\|\int_{t_T}^{t}\ddot{p}_h dl\|^2 + 2k_1^2 j_{p_i}^2\|\int_{t_T}^{t}\frac{d}{dl}(\phi(e_p)e_p dl\|^2$$

the following upper bound can be developed

$$\begin{aligned} &\|\eta\|\|(W_y - W_{y_T})\zeta_y\| \leq \bar{m}\bar{j}_{p_{inv}}\bar{p}_3\|\eta\| \\ &+ (\frac{(k_1+1)\bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}\gamma_7^2 + \bar{c}\gamma_9^2}{4}\|\dot{\theta}\|^2)\|\eta\|^2 \\ &+ (\frac{\bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}}{\gamma_7^2} + \frac{\bar{c}2j_{p_i}^2}{\gamma_9^2})\|\int_{t_T}^{t}\ddot{p}_h(l)dl\|^2 \\ &+ k_1\bar{m}\bar{j}_{p_{inv}}\|\eta\|\|\dot{\phi}(e_p)e_p - \dot{\phi}(e_{p_T})e_{p_T} \\ &+ \phi(e_p)\dot{e}_p - \phi(e_{p_T})\dot{e}_{p_T}\| \\ &+ (\frac{k_1\bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}}{\gamma_7^2} + \frac{\bar{c}2k_1^2j_{p_i}^2}{\gamma_9^2})\|\int_{t_T}^{t}\frac{d}{dl}\phi(e_p)(l)e_p(l)dl\|^2 \end{aligned} \quad (4)$$

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To further develop the bound in (4), following bounds are developed. Using the triangle inequality of vector norms results in

$$\begin{aligned}\|\dot{\phi}(e_p)e_p - \dot{\phi}(e_{p_T})e_{p_T}\| \leq &\|2k_1(\phi^3(e_p)e_p^2 - \phi^3(e_{p_T})e_{p_T}^2)\| \\ &+ \|2(\phi^2(e_p)e_p - \phi^2(e_{p_T})e_{p_T})\|\|W_j\tilde{\zeta}_j\| \\ &+ \|2\phi^2(e_p)e_p\hat{J}_p\eta - 2\phi^2(e_{p_T})e_{p_T}\hat{J}_p\eta_T\|\end{aligned}$$

where the symbol $\phi^3(e_p)e_p^2 = [\phi^3(e_{p1})e_{p1}^2, ..., \phi^3(e_{pn})e_{pn}^2]^T$ is a vector containing elements $\phi^3(e_{pi})e_{pi}^2$ and $\phi^2(e_p)e_p$ is a diagonal matrix containing elements $\phi^2(e_{pi})e_{pi}$. Using Appendix A of [1], bounds on each term can be developed as

$$\begin{aligned}&\|2k_1(\phi^3(e_p)e_p^2 - \phi^3(e_{p_T})e_{p_T}^2)\| \leq \rho_2(\|e_p\|)\|p_h - p_{h_T}\| \\ &\|2(\phi^2(e_p)e_p - \phi^2(e_{p_T})e_{p_T})\| \leq \rho_3(\|e_p\|)\|p_h - p_{h_T}\| \\ &\|2\phi^2(e_p)e_p\hat{J}_p\eta - 2\phi^2(e_{p_T})e_{p_T}\hat{J}_p\eta_T\| \\ &\leq \rho_4(\|z_1\|)\|z_1(t) - z_1(t-T)\|\end{aligned} \quad (5)$$

where ρ_i, $\forall i = \{2, 3, 4\}$ are positive invertible functions, $z_1(t) = [e_p^T \ \eta^T \ ]^T \in \mathcal{D} \subset \mathbb{R}^{2n}$ and $z_1(t) - z_1(t-T) = [(p_h - p_{h_T})^T \ (\eta - \eta_T)^T \ ]^T$.

$$\begin{aligned}&\|\eta\|\|\dot{\phi}(e_p)e_p - \dot{\phi}(e_{p_T})e_{p_T}\| \leq \|\eta\|\rho_2(\|e_p\|)\|p_h - p_{h_T}\| \\ &+ \|\eta\|\rho_3(\|e_p\|)\|p_h - p_{h_T}\|\|\|W_j\tilde{\zeta}_j\| + \rho_4(\|z_1\|)\|\eta\|\|\eta - \eta_T\| \\ &+ \rho_4(\|z_1\|)\|\eta\|\|p_h - p_{hT}\|\end{aligned} \quad (6)$$

Using Young's inequality

$$\begin{aligned}&\|\eta\|\|\dot{\phi}(e_p)e_p - \dot{\phi}(e_{p_T})e_{p_T}\| \leq \rho_2(\|e_p\|)\|\eta\|\|p_h - p_{h_T}\| \\ &+ \rho_4(\|z_1\|)\|\eta\|\|p_h - p_{hT}\| \\ &+ \frac{\bar{p}_1 w_j \rho_3^2(\|e_p\|)\|\dot{\theta}\|^2\gamma_{11}^2 + \rho_4^2(\|z_1\|)\gamma_{12}^2}{4}\|\eta\|^2 + \frac{\bar{p}_1 w_j}{\gamma_{11}^2}\|\tilde{\zeta}_j\|^2 \\ &+ \frac{2j_{p_i}^2}{\gamma_{12}^2}\|\int_{t_T}^t \ddot{p}_h(l)dl\|^2 + \frac{2k_1^2 j_{p_i}^2}{\gamma_{12}^2}\|\int_{t_T}^t \frac{d}{dl}(\phi(e_p)(l)e_p(l))dl\|^2\end{aligned} \quad (7)$$

Consider following term from (4)

$$\begin{aligned}&\|\phi(e_p)\dot{e}_p - \phi(e_{p_T})\dot{e}_{p_T}\| \leq \|k_1(\phi^2(e_p)e_p - \phi^2(e_{p_T})e_{p_T})\| \\ &+ \|\phi(e_p) - \phi(e_{p_T})\|\|W_j\tilde{\zeta}_j\| + \|\phi(e_p)\hat{J}_p\eta - \phi(e_{p_T})\hat{J}_p\eta_T\|\end{aligned}$$

where $\phi(e_p)$ is a diagonal matrix containing terms $\phi(e_{pi})$, $\phi^2(e_p)e_p$ is a vector containing terms $\phi^2(e_{pi})e_{pi}$. Using Appendix A of [1], each term can be norm bounded as

$$\begin{aligned}&\|k_1(\phi^2(e_p)e_p - \phi^2(e_{p_T})e_{p_T})\| \leq \rho_5(\|e_p\|)\|p_h - p_{h_T}\| \\ &\|(\phi(e_p) - \phi(e_{p_T}))\| \leq \rho_6(\|e_p\|)\|p_h - p_{h_T}\| \\ &\|\phi(e_p)\hat{J}_p\eta - \phi(e_{p_T})\hat{J}_p\eta_T\| \leq \rho_7(\|z_1\|)\|z_1(t) - z_1(t-T)\|\end{aligned}$$

such that ρ_i, $\forall i = \{5, 6, 7\}$ are positive invertible functions. The norm bound can be written as

$$\begin{aligned}&\|\eta\|\|\phi(e_p)\dot{e}_p - \phi(e_{p_T})\dot{e}_{p_T}\| \leq \rho_5(\|e_p\|)\|\eta\|\|p_h - p_{h_T}\| \\ &+ \bar{p}_1 w_j \rho_6(\|e_p\|)\|\dot{\theta}\|\|\eta\|\|\tilde{\zeta}_j\| + \rho_7(\|z_1\|)\|\eta\|\|\eta - \eta_T\| \\ &+ \rho_7(\|z_1\|)\|\eta\|\|p_h - p_{hT}\|\end{aligned} \quad (8)$$

Using Young's Inequality,

$$\begin{aligned}&\|\eta\|\|\phi(e_p)\dot{e}_p - \phi(e_{p_T})\dot{e}_{p_T}\| \leq \rho_5(\|e_p\|)\|\eta\|\|p_h - p_{h_T}\| \\ &+ \rho_7(\|z_1\|)\|\eta\|\|p_h - p_{hT}\| \\ &+ \frac{\bar{p}_1 w_j \rho_6^2(\|e_p\|)\|\dot{\theta}\|^2\gamma_{13}^2 + \rho_7^2(\|z_1\|)\gamma_{14}^2}{4}\|\eta\|^2 + \frac{\bar{p}_1 w_j}{\gamma_{13}^2}\|\tilde{\zeta}_j\|^2 \\ &+ \frac{2j_{p_i}^2}{\gamma_{14}^2}\|\int_{t_T}^t \ddot{p}_h(l)dl\|^2 + \frac{2k_1^2 j_{p_i}^2}{\gamma_{14}^2}\|\int_{t_T}^t \frac{d}{dl}(\phi(e_p)(l)e_p(l))dl\|^2\end{aligned} \quad (9)$$

Using (7) and (9), the bound in (4) can be written as

$$\begin{aligned}&\|\eta\|\|(W_y - W_{y_T})\zeta_y\| \leq \bar{m}\bar{j}_{p_{inv}}\bar{p}_3\|\eta\| + \rho_9(\|z_1\|, \|\dot{\theta}\|)\|\eta\|^2 \\ &+ c_{LK_1}\|\int_{t_T}^t \ddot{p}_h(l)dl\|^2 + c_{LK_2}\|\int_{t_T}^t \frac{d}{dl}\phi(e_p)(l)e_p(l)dl\|^2 \\ &+ c_{LK_3}\|\int_{t_t}^t \dot{p}_h dl\|^2 + (\frac{\bar{p}_1 w_j}{\gamma_{11}^2} + \frac{\bar{p}_1 w_j}{\gamma_{13}^2})\|\tilde{\zeta}_j\|^2\end{aligned} \quad (10)$$

where $\rho_8(\|z_1\|) = \rho_2(\|e_p\|) + \rho_5(\|e_p\|) + \rho_4(\|z_1\|) + \rho_7(\|z_1\|)$,$\rho_9(\|z_1\|, \|\dot{\theta}\|) = (\frac{(k_1+1)\bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}\gamma_7^2 + \bar{c}\gamma_9^2 + \bar{p}_1 w_j \rho_3^2(\|e_p\|)\|\dot{\theta}\|^2\gamma_{11}^2 + \rho_4^2(\|z_1\|)\gamma_{12}^2}{4}\|\dot{\theta}\|^2 + \frac{\bar{p}_1 w_j \rho_6^2(\|e_p\|)\|\dot{\theta}\|^2\gamma_{13}^2 + \rho_7^2(\|z_1\|)\gamma_{14}^2}{4} + \frac{\gamma_{15}^2\rho_8^2(\|z_1\|)}{4})$,$c_{LK_1} = (\frac{\bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}}{\gamma_7^2} + \frac{\bar{c}2j_{p_i}^2}{\gamma_9^2} + \frac{2j_{p_i}^2}{\gamma_{12}^2} + \frac{2j_{p_i}^2}{\gamma_{14}^2})$, $c_{LK_2} = (\frac{k_1\bar{m}\bar{j}_{p_{inv}}^2\bar{j}_{p_{der}}}{\gamma_7^2} + \frac{\bar{c}2k_1^2 j_{p_i}^2}{\gamma_9^2} + \frac{2k_1^2 j_{p_i}^2}{\gamma_{12}^2} + \frac{2k_1^2 j_{p_i}^2}{\gamma_{14}^2})$, $c_{LK_3} = \frac{1}{\gamma_{15}^2}$.

APPENDIX C
DEVELOPMENT OF BOUNDS ON CROSS TERMS

Using Cauchy-Schwarz, and Young's inequality and $\|W_j\| \leq w_j\|\dot{\theta}\|$

$$\begin{aligned}\|\tilde{\zeta}_j^T W_j^T \int \frac{d}{dt}\phi(e_p)e_p dl\| \leq &\frac{\gamma_6^2}{4}w_j^2\|\dot{\theta}\|^2\|\tilde{\zeta}_j\|^2 \\ &+ \frac{1}{\gamma_6^2}\|\int \frac{d}{dt}\phi(e_p)e_p dl\|^2\end{aligned} \quad (11)$$

where $\gamma_6 \in \mathbb{R}^+$ is a constant. Similarly using Cauchy-Schwarz, Young's inequality and $\|W_{y_T}\| \leq \rho_2(\|y_T\|) = w_{y_T}\|y_T\| + \bar{w}_{y_T}$, where $y_T = [e_{p_T}^T \ \eta_T^T \ \tilde{\zeta}_j^T \ ]^T \in \mathcal{D} \subset \mathbb{R}^{2n+m_1}$ and $\|W_y\| \leq w_y\|y\| + \bar{w}_y$, we have

$$\begin{aligned}\|\tilde{\zeta}_y^T W_{y_T}^T(\eta - \eta_T)\| \leq &\frac{\gamma_5^2}{4}\|\tilde{\zeta}_y\|^2 + \frac{2j_{p_i}^2}{\gamma_5^2}\rho_{10}^2(\|y_T\|) \\ &\|\int_{t_T}^t \ddot{p}_h(l)dl\|^2 + \frac{2k_1^2 j_{p_i}^2}{\gamma_5^2}\rho_{10}^2(\|y_T\|) \\ &\|\int_{t_T}^t \frac{d}{dt}(\phi(e_p)(l)e_p(l))dl\|^2\end{aligned} \quad (12)$$

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